Set - II Std : 12th HSC	Subject – Maths	Model Answer

Section A

Q.1 Select and write the correct answer

(i) Answer: **d)** $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6log\left|\sqrt[6]{x}+1\right| + c$

$I = \int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

Let $x = t^6, dx = 6t^5 dt$

$\therefore I = \int \frac{6t^5 dt}{t^3+t^2} = 6\int \frac{t^3}{t+1}dt = 6\int \frac{t^3+1-1}{t+1}dt$

$= 6\int \left[\frac{(t+1)(t^2-t+1)}{t+1} - \frac{1}{t+1}\right]dt = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - log|t+1|\right] + c$

$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6log\left|\sqrt[6]{x}+1\right| + c$

(ii) Answer: **a)** $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = iI, A^2 = i^2 I = -I$

$\therefore A^4 = I, \forall n \in N, A^{4n} = I$

(iii) Answer**: c)** $\boldsymbol{x + y + 4 = 0}$

$y^2 = 16x$

Differentiating w.r.t.x

$2y\frac{dy}{dx} = 16, \frac{dy}{dx} = \frac{8}{y}$,slope of the tangent at $(4,-8)$ is $\frac{8}{y} = \frac{8}{-8} = -1$

Equation of the tangent is $y + 8 = -1(x-4), \therefore x + y + 4 = 0$

(iv) Answer: **b)** $\boldsymbol{-\frac{35}{4}, -5}$

$12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$

Here $a = 12, 2h = 2k, b = 2, 2g = 11, 2f = -5, c = 2$

Equation represents a pair of lines

$\therefore \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0, \therefore \begin{vmatrix} 12 & k & \frac{11}{2} \\ k & 2 & -\frac{5}{2} \\ \frac{11}{2} & -\frac{5}{2} & 2 \end{vmatrix} = 0$

$\therefore 12\left[4 - \frac{25}{4}\right] - k\left[2k + \frac{55}{4}\right] + \frac{11}{2}\left[-\frac{5k}{2} - 11\right] = 0$

$\therefore 48 - 75 - 2k^2 - \frac{55k}{4} - \frac{55k}{4} - \frac{121}{2} = 0$

$\therefore 4k^2 + 55k + 175 = 0, \therefore 4k^2 + 20k + 35k + 175 = 0,$

$\therefore 4k(k+5) + 25(k+5) = 0, \therefore (k+5)(4k+35) = 0$

$\boldsymbol{\therefore k = -5, -\frac{35}{4}}$

(v) Answer: **d)** $\boldsymbol{\frac{1}{2\sqrt{x}}\left(\frac{3}{1+9x} - \frac{1}{1+x}\right)}$

$y = \tan^{-1}\left(\frac{2\sqrt{x}}{1+3x}\right) = \tan^{-1}\left[\frac{3\sqrt{x}-\sqrt{x}}{1+3\sqrt{x}.\sqrt{x}}\right] = \tan^{-1}3\sqrt{x} - \tan^{-1}\sqrt{x}$

Differentiating w.r.t.x

$\frac{dy}{dx} = \frac{3}{2\sqrt{x}(1+9x)} - \frac{1}{2\sqrt{x}(1+x)} = \frac{1}{2\sqrt{x}}\left(\frac{3}{1+9x} - \frac{1}{1+x}\right)$

(vi) Answer: **c)** $\boldsymbol{p = 3}$

Since the vectors are coplanar,

$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -5 & p \\ 5 & -9 & 4 \end{vmatrix} = 0, \therefore 1(-20+9p) + 2(8-5p) + 1(-18+25) = 0$

$\therefore -20 + 9p + 16 - 10p + 7 = 0, \therefore -p = -3, \therefore \boldsymbol{p = 3}$

(vii) Answer: **c)** $\boldsymbol{p \wedge q}$

$\sim(p \to \sim q) \equiv \sim(\sim p \vee \sim q) \equiv p \wedge q$

(viii) Answer: **d) 4.04**

$\sqrt[3]{66} = (64+2)^{\frac{1}{3}}$

Here $a = 64, h = 2, f(x) = x^{\frac{1}{3}}, f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f(a) = f(64) = 4, f'(a) = f'(64) = \frac{1}{48}$

$f(a+h) \approx f(a) + hf'(a)$

$\therefore \sqrt[3]{66} = 4 + 2\left(\frac{1}{48}\right) = 4 + \frac{1}{24} = 4.04$

Q.2 Answer the following

(i) $\frac{d^3y}{dx^3} = \left[1 + \left(\frac{d^2y}{dx^2}\right)^3\right]^2$ **.Order is 3 and degree is 1**

(ii) In ΔABC, $a = 12, b = 5, c = 13$

$a^2 + b^2 = 144 + 25 = 169 = c^2$

$\therefore$ABC is a right-angled triangle.

(iii) Here,$E(X) = 18 = np, V(X) = 12 = npq$

$\frac{npq}{np} = \frac{12}{18}, \therefore q = \frac{2}{3}, p = 1 - q = \frac{1}{3}$

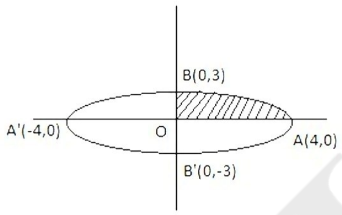
$\therefore np = 18, \frac{1}{3}n = 18, \therefore \boldsymbol{n = 54}$

(iv) Let p:I drive fast : I follow traffic rules,r : I will surely meet with an accident. Symbolic form: $\boldsymbol{(p \wedge \sim q) \to r}$

Section B

Attempt any Eight

Q.3



$\frac{x^2}{16} + \frac{y^2}{9} = 1, \therefore y^2 = 9\left[1 - \frac{x^2}{16}\right], \therefore y = \frac{3}{4}\sqrt{16 - x^2}$

Area of the ellipse = $4\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx$

$= 3\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$\ldots .using \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

$\therefore$ Area of the ellipse $= 3\left[0 + 8\left(\frac{\pi}{2}\right) - (0 + 0)\right] = \boldsymbol{12\pi sq.units}$

Q.4 $x = 3sin\theta - 2sin^3\theta, y = 3cos\theta - 2cos^3\theta$

Differentiating w.r.t.'θ'

$\frac{dx}{d\theta} = 3cos\theta - 6sin^2\theta cos\theta = 3cos\theta(1 - 2sin^2\theta) = 3cos\theta cos2\theta$

$\frac{dy}{d\theta} = -3sin\theta + 6cos^2\theta sin\theta = 3sin\theta(2cos^2\theta - 1) = 3sin\theta cos2\theta$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{3sin\theta cos2}{3cos\theta cos2\theta} = tan\theta$

When $\theta = \frac{\pi}{4}, \frac{dy}{dx} = tan\frac{\pi}{4} = 1$.Hence proved.

Q.5 Cartesian form is $2x + 3y - z = 7$

Vector form is $\bar{\boldsymbol{r}}.\left(\boldsymbol{2\bar{\imath} + 3\bar{\jmath} - \bar{k}}\right) = \boldsymbol{7}$

Q.6 Let $\bar{a} = \bar{\imath} + \bar{\jmath} + \bar{k}, \bar{b} = 2\bar{\imath} - \bar{\jmath} + \bar{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\bar{\imath} + \bar{\jmath} - 3\bar{k}$$

∴dr's of the line are 2,1, -3

Vector equation: $\bar{\boldsymbol{r}} = \left(\bar{\boldsymbol{\imath}} + \boldsymbol{2}\bar{\boldsymbol{\jmath}} + \boldsymbol{3}\bar{\boldsymbol{k}}\right) + \boldsymbol{\lambda}\left(\boldsymbol{2}\bar{\boldsymbol{\imath}} + \bar{\boldsymbol{\jmath}} - \boldsymbol{3}\bar{\boldsymbol{k}}\right)$

Cartesian form: $\boldsymbol{\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-3}}$

Q.7 $\overline{AB} = \bar{\imath} + 5\bar{\jmath} - 7\bar{k}$, $\overline{BC} = -2\bar{\imath} - 10\bar{\jmath} + 14\bar{k} = -2\left(\bar{\imath} + 5\bar{\jmath} - 7\bar{k}\right) = -2\overline{AB}$

And point B is common. ∴ Points A,B and C are collinear.

Q.8 $cosx = -\frac{\sqrt{3}}{2}, cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ and $cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\therefore \boldsymbol{x = \frac{5\pi}{6}, \frac{7\pi}{6}}$

Q.9 $xe^{2y} + ye^{2x} = 1$

Differentiating w.r.t.x

$e^{2y} + x(2e^{2y})\frac{dy}{dx} + 2ye^{2x} + e^{2x}.\frac{dy}{dx} = 1$

$\therefore (2xe^{2y} + e^{2x})\frac{dy}{dx} = -e^{2y} - 2ye^{2x}$

$\therefore \boldsymbol{\frac{dy}{dx} = -\left(\frac{e^{2y}+2ye^{2x}}{2xe^{2y}+e^{2x}}\right)}$

Q.10 Let $\bar{a} = 3\bar{\imath} - 5\bar{\jmath} + \bar{k}, \bar{b} = 5\bar{\imath} + 4\bar{\jmath} + 2\bar{k}, \bar{c} = 7\bar{\imath} - 7\bar{\jmath} + 3\bar{k}, \bar{d} = \bar{\imath} + 2\bar{k}$ be the position vectors of the points A,B,C,D.

Centroid,$\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4} = \frac{3\bar{\imath}-5\bar{\jmath}+\bar{k}+5\bar{\imath}+4\bar{\jmath}+2\bar{k}+7\bar{\imath}-7\bar{\jmath}+3\bar{k}+\bar{\imath}+2\bar{k}}{4} = \frac{16\bar{\imath}-8\bar{\jmath}+8\bar{k}}{4}$

$= \boldsymbol{4\bar{\imath} - 2\bar{\jmath} + 2\bar{k}}$. Centroid is $\boldsymbol{(4, -2, 2)}$

Q.11 $I = \int \frac{x+sinx}{1-cosx}dx = \int \frac{x+2sin\frac{x}{2}cos\frac{x}{2}}{2sin^2\frac{x}{2}}dx = \frac{1}{2}\int xcosec^2\frac{x}{2}dx + \int cot\frac{x}{2}dx$

$= \frac{1}{2}\left[-x2cot\left(\frac{x}{2}\right) + \int 1.2cot\left(\frac{x}{2}\right)dx\right] + 2log\left|sin\left(\frac{x}{2}\right)\right| + c$

$= -xcot\left(\frac{x}{2}\right) + 2log\left|sin\left(\frac{x}{2}\right)\right| + 2log\left|sin\left(\frac{x}{2}\right)\right| + c$

$= \boldsymbol{-xcot\left(\frac{x}{2}\right) + 4log\left|sin\left(\frac{x}{2}\right)\right| + c}$

Q.12 $I = \int_0^1 \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Let $x = tan\theta, dx = sec^2\theta d\theta$

When $x = 0, \theta = 0, x = 1, \theta = \frac{\pi}{4}$

$\therefore I = \int_0^{\frac{\pi}{4}} \cos^{-1}(cos2\theta)sec^2\theta d\theta = \int_0^{\frac{\pi}{4}} 2\theta sec^2\theta d\theta$

$= 2[\theta tan\theta - log|sec\theta|]_0^{\frac{\pi}{4}} = 2\left[\frac{\pi}{4} - log\sqrt{2}\right] = \boldsymbol{\frac{\pi}{2} - log2}$

Q.13 (a)12 is a prime number.

It is a statement. Truth value : F

(b)$x^2 - 3x - 4 = 0$

It is not a statement.

Q.14 $f(x) = \frac{x}{8} \qquad 0 < x < 4$

$\qquad = 0 \qquad otherwise.$

$P(X \leq 2) = \int_0^2 \frac{x}{8}dx = \left[\frac{x^2}{16}\right]_0^2 = \boldsymbol{\frac{1}{4}}$

$P(X > 3) = \int_3^4 \frac{x}{8}dx = \left[\frac{x^2}{16}\right]_3^4 = \frac{16}{16} - \frac{9}{16} = \boldsymbol{\frac{7}{16}}$

Find $P(X \leq 2), P(X > 3)$

Section C

Attempt any Eight

Q.15 $cos^2(x^2+y^2) = x + y\frac{dy}{dx}$

Let $x^2+y^2 = v, \therefore 2x + 2y\frac{dy}{dx} = \frac{dv}{dx}, \therefore x + y\frac{dy}{dx} = \frac{1}{2}\frac{dv}{dx}$

Equation is $cos^2 v = \frac{1}{2}\frac{dv}{dx}, \therefore 2dx = sec^2 v dv$

Integrating, $2x = tanv + c$

$\therefore 2x = tan(x^2+y^2) + c$

when x = 0, y = 0

$\therefore 0 = tan0 + c, \therefore c = 0$

Particular solution is $\boldsymbol{2x = tan(x^2+y^2)}$

Q.16 $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

Using, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \frac{\pi}{4}, \therefore \frac{5x}{1-6x^2} = 1, \therefore 6x^2+5x-1 = 0, \therefore (x+1)(6x-1) = 0, \therefore x = -1, \frac{1}{6}$

$x \neq -1, \therefore \boldsymbol{x = \frac{1}{6}}$

Q.17 Let $\bar{a}$ and $\bar{b}$ be the pvs of the points A and B w.r.t.a fixed origin. Let C$(\bar{c})$ divide seg AB in the ratio 2 : 5 externally

$\bar{a} = 3\bar{\imath} + 4\bar{\jmath} + 3\bar{k}, \bar{b} = 2\bar{\imath} + 2\bar{\jmath} + 7\bar{k}$

Using section formula for external division,

$\bar{c} = \frac{2\bar{b}-3\bar{a}}{2-5} = \frac{2(2\bar{\imath}+2\bar{\jmath}+7\bar{k})-5(3\bar{\imath}+4\bar{\jmath}+3\bar{k})}{-3} = \frac{\boldsymbol{-11\bar{\imath}-16\bar{\jmath}-\bar{k}}}{\boldsymbol{-3}} = \frac{\boldsymbol{11\bar{\imath}+16\bar{\jmath}+\bar{k}}}{\boldsymbol{3}}$

$\boldsymbol{C\left(\frac{11}{3}, \frac{16}{3}, \frac{1}{3}\right)}$

Q.18 P(1,2,-1),Q(8,-3,-4),R(5,-4,1) and S(-2,1,4)

Let $\bar{p}, \bar{q}, \bar{r}, \bar{s}$ be the pvs of the points P,Q,R,S w.r.t. some fixed origin.

$\bar{p} = \bar{\imath} + 2\bar{\jmath} - \bar{k}, \bar{q} = 8\bar{\imath} - 3\bar{\jmath} - 4\bar{k}$

$\bar{r} = 5\bar{\imath} - 4\bar{\jmath} + \bar{k}, \bar{s} = -2\bar{\imath} + \bar{\jmath} + 4\bar{k}$

$\therefore \overline{PQ} = \bar{q} - \bar{p} = 7\bar{\imath} - 5\bar{\jmath} - 3\bar{k}, \overline{SR} = \bar{r} - \bar{s} = 7\bar{\imath} - 5\bar{\jmath} - 3\bar{k}$

$\therefore \overline{PQ} = \overline{SR}$

$\therefore PQ \parallel SR$ and $|\overline{PQ}| = |\overline{SR}|$

∴Quadrilateral PQRS is a parallelogram.

Q.19 $3x - 1 = 6y + 2 = 2z - 3$

$\frac{3\left(x-\frac{1}{3}\right)}{6} = \frac{6\left(y+\frac{1}{3}\right)}{6} = \frac{2\left(z-\frac{3}{2}\right)}{6}$

$\frac{\left(x-\frac{1}{3}\right)}{2} = \frac{\left(y+\frac{1}{3}\right)}{1} = \frac{\left(z-\frac{3}{2}\right)}{3}$

drs are 2,1,3

∴dcs are $\boldsymbol{\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}}$

Equation of a line parallel to $3x - 1 = 6y + 2 = 2z - 3$ and through point (2,1,3) is

$\boldsymbol{\frac{x-2}{2} = \frac{y-1}{1} = \frac{z-3}{3}}$

Q.20

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

(a)Since $P(x)$ is the probability distribution of 'x',

$\sum_{x=0}^{6} P(x) = 1, \therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1, \therefore 49k = 1 \therefore k = \boldsymbol{\frac{1}{49}}$

(b) $P(X \geq 2) = 5k + 7k + 9k + 11k + 13k = 45k = \boldsymbol{\frac{45}{49}}$

(c) c.d.f. of X.

x_i	0	1	2	3	4	5	6
$\boldsymbol{F(x_i)}$	$\boldsymbol{\frac{1}{49}}$	$\boldsymbol{\frac{4}{49}}$	$\boldsymbol{\frac{9}{49}}$	$\boldsymbol{\frac{16}{49}}$	$\boldsymbol{\frac{25}{49}}$	$\boldsymbol{\frac{36}{49}}$	**1**

Q.21 $y = \left(x + \sqrt{x^2 - 1}\right)^m$

Differentiating w.r.t.'x'

$\frac{dy}{dx} = m\left(x + \sqrt{x^2 - 1}\right)^{m-1}\left[1 + \frac{2x}{2\sqrt{x^2-1}}\right]$

$= m\left(x + \sqrt{x^2 - 1}\right)^{m-1}\left[\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right]$

$\frac{dy}{dx} = m\frac{\left(x+\sqrt{x^2-1}\right)^m}{\sqrt{x^2-1}}, \therefore \sqrt{x^2 - 1}\frac{dy}{dx} = my$ …(1)

Differentiating w.r.t.'x'

$\sqrt{x^2 - 1}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{x^2-1}}\frac{dy}{dx} = m\frac{dy}{dx}$

$\therefore (x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m\sqrt{x^2 - 1}\frac{dy}{dx}$

From (1)

$(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m(my)$

$\therefore (x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m^2y$…hence proved.

Q.22 Let the dimensions of the box be x,x,y.

Surface area of the box = $x^2 + 4xy = 192$

$\therefore y = \frac{192-x^2}{4x}$

Volume of the box = $x^2y = x^2\left(\frac{192-x^2}{4x}\right) = x\left(48 - \frac{x^2}{4}\right) = 48x - \frac{x^3}{4}$

Let $V(x) = 48x - \frac{x^3}{4}, \therefore V'(x) = 48 - \frac{3x^2}{4}$

For maximum volume, $V'(x) = 0, \therefore 48 - \frac{3x^2}{4} = 0, \therefore x = 8$

$\therefore y = 4$

$V''(x) = -\frac{3x}{2}, \therefore V''(8) = -8 < 0$

∴Volume is maximum when the dimensions are **8cm,8cm,4cm**

Q.23 $I = \int \frac{1}{4+5sinx}dx$

Let $tan\frac{x}{2} = t, \therefore dx = \frac{2dt}{1+t^2}$

$\therefore I = \int \frac{2dt/_{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)} = \int \frac{2dt}{4+4t^2+10t} = \int \frac{dt}{2t^2+5t+2} = \frac{1}{2}\int \frac{dt}{t^2+\frac{5}{2}t+1}$

$= \frac{1}{2}\int \frac{dt}{t^2+\frac{5}{2}t+\frac{25}{16}+1-\frac{25}{16}} = \frac{1}{2}\int \frac{dt}{\left(t+\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}$

Using $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}log\left|\frac{x-a}{x+a}\right| + c$

$\therefore I = \frac{1}{2}.\frac{1}{2\left(\frac{3}{4}\right)}log\left|\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right| + c = \frac{1}{3}log\left|\frac{2t+1}{2t+4}\right| + c$

$= \boldsymbol{\frac{1}{3}log\left|\frac{2tan\frac{x}{2}+1}{2tan\frac{x}{2}+4}\right| + c}$

Q.24 To prove:

$I = \int uv\,dx = u\int vdx - \int\left[\frac{du}{dx}\int vdx\right]dx$

Let $\int vdx = w, \therefore \frac{dw}{dx} = v$

Consider $\frac{d}{dx}(uw) = u\frac{dw}{dx} + w\frac{du}{dx}$

Integrating both sides w.r.t.x,

$uw = \int\left[u\frac{dw}{dx} + w\frac{du}{dx}\right]dx$, $\therefore u\int vdx = \int uvdx + \int(\int vdx)\left(\frac{du}{dx}\right)dx$

$\therefore \int uvdx = u\int vdx - \int\left[\frac{du}{dx}\int vdx\right]dx$, hence proved.

$I = \int xsinxdx = x\int sinxdx - \int\left(\frac{d}{dx}x\right)(\int sinxdx)dx$

$= -xcosx + \int cosxdx = \boldsymbol{-xcosx + sinx + c}$

Q.25 $[(p \vee q) \vee (p \wedge r)] \rightarrow \sim p$

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$ (1)	$\sim p$ (2)	(1)→(2)
T	T	T	T	T	T	F	**F**
T	T	F	T	F	T	F	**F**
T	F	T	T	T	T	F	**F**
T	F	F	T	F	T	F	**F**
F	T	T	T	F	T	T	**T**
F	T	F	T	F	T	T	**T**
F	F	T	F	F	F	T	**T**
F	F	F	F	F	F	T	**T**

Q.26 Let the foot of the perpendicular be $P(x_1, y_1, z_1)$.Origin O(0,0,0)

Drs of OM are x_1, y_1, z_1

Drs of normal to the plane are 2,-1,-2

OM ∥ normal,$\therefore \frac{x_1}{2} = \frac{y_1}{-1} = \frac{z_1}{-2} = t$

$\therefore x_1 = 2t, y_1 = -t, z_1 = -2t$

M lies on the plane.

$\therefore 2(2t) - (-t) - 2(-2t) = 27, \therefore t = 3, \therefore \boldsymbol{M = (6, -3, -6)}$

Section D

Attempt any five

Q.27 $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 6 & 2 \\ 7 & 0 & -1 \\ -7 & -3 & 2 \end{bmatrix}$

$XA = B, \therefore X = BA^{-1}$

$|A| = 1(-2) + 2(-2) - 1(-2) = -4$

Minors :

$M_{11} = -2, M_{12} = -2, M_{13} = -2, M_{21} = 1, M_{22} = 3, M_{23} = 7, M_{31} = -1, M_{32} = 1, M_{33} = 1$

Co-factors :

$A_{11} = -2, A_{12} = 2, A_{13} = -2, A_{21} = -1, A_{22} = 3, A_{23} = -7, A_{31} = -1, A_{32} = -1, A_{33} = 1$

adjointA = $\begin{bmatrix} -2 & -1 & -1 \\ 2 & 3 & -1 \\ -2 & -7 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adjoint\ A = \frac{1}{-4} \begin{bmatrix} -2 & -1 & -1 \\ 2 & 3 & -1 \\ -2 & -7 & 1 \end{bmatrix}$

$X = BA^{-1} = \frac{1}{-4} \begin{bmatrix} 8 & 6 & 2 \\ 7 & 0 & -1 \\ -7 & -3 & 2 \end{bmatrix} \begin{bmatrix} -2 & -1 & -1 \\ 2 & 3 & -1 \\ -2 & -7 & 1 \end{bmatrix} = \frac{1}{-4} \begin{bmatrix} -8 & -4 & -12 \\ -12 & 0 & -8 \\ 4 & -16 & 12 \end{bmatrix} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{3} & \mathbf{0} & \mathbf{2} \\ \mathbf{-1} & \mathbf{4} & \mathbf{-3} \end{bmatrix}$

Q.28 Let m_1 and m_2 be the slopes of the two lines given by $3x^2 + 4xy - 3y^2 = 0$.

Here a = 3,2h = 4,b = -3.$m_1 + m_2 = -\frac{2h}{b} = \frac{4}{3}$ and $m_1 m_2 = \frac{a}{b} = \frac{3}{-3} = -1$.The slopes of the lines perpendicular to the given pair of lines are $\frac{-1}{m_1}$&$\frac{-1}{m_2}$. Since they pass through the origin their equations are,$y = -\frac{1}{m_1}x$&$y = -\frac{1}{m_2}x$, $i.e.\ x + m_1 y = 0$ & $x + m_2 y = 0$

Combined equation is $(x + m_1 y)(x + m_2 y) = 0$,

i.e.$x^2 + (m_1 + m_2)xy + m_1 m_2 y^2 = 0$

$\therefore x^2 + \frac{4}{3}xy - y^2 = 0$,i.e. $\mathbf{3x^2 + 4xy - 3y^2 = 0}$ is the required equation.

Q.29 Let X = success in making a telephone call

n = 10,p = 0.8,q = 0.2,x = 7

$P(X = x) = n_{C_x}(p^x)(q^{n-x})$,$\therefore P(X = 7) = 10_{C_7}(0.8)^7(0.2)^3$

$\therefore P(X = 7) = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} \times \left(\frac{4}{5}\right)^7 \times \left(\frac{1}{5}\right)^3 = \mathbf{0.2013}$

Q.30 Let the amount of ice at any instant 't' be A.

$\frac{dA}{dt} = -kt, \therefore \frac{dA}{A} = -kdt, \therefore logA = -kt + c$ …….(1)

When $t = 0, let\ A = A_0$

$\therefore logA_0 = -k(0) + c, \therefore c = logA_0$

In (1), $logA = -kt + logA_0, \therefore log\left(\frac{A}{A_0}\right) = -kt$

When $t = 20\ minutes, A = \frac{1}{2}A_0$

In (2), $log\left(\frac{\frac{1}{2}A_0}{A_0}\right) = -k(20), \therefore k = -\frac{1}{20}log\left(\frac{1}{2}\right)$

$\therefore log\left(\frac{A}{A_0}\right) = \frac{t}{20}log\left(\frac{1}{2}\right)$

When $t = 1hour = 60\ minutes$

$log\left(\frac{A}{A_0}\right) = \frac{60}{20}log\left(\frac{1}{2}\right), \therefore \frac{A}{A_0} = \left(\frac{1}{2}\right)^3, \therefore A = \frac{1}{8}A_0$

Hence proved.

Q.31 $f(x) = 2x + \frac{1}{2x}$

$f'(x) = 2 - \frac{1}{2x^2}$

(a) increasing, $f'(x) > 0, 2 - \frac{1}{2x^2} > 0, 4x^2 > 1, x > \frac{1}{2}\ or\ x < -\frac{1}{2}$. Solution is $\boldsymbol{x} \in \left(-\infty, -\frac{\mathbf{1}}{\mathbf{2}}\right) \cup \left(\frac{\mathbf{1}}{\mathbf{2}}, \infty\right)$

(b) decreasing, $f'(x) < 0, 2 - \frac{1}{2x^2} < 0, 4x^2 < 1, x > -\frac{1}{2}\ or\ x < \frac{1}{2}$. Solution is $-\frac{\mathbf{1}}{\mathbf{2}} < x < \frac{\mathbf{1}}{\mathbf{2}}$

Q.32 Consider,R.H.S. $= \int_a^b f(a + b - x)dx$

Let $a + b - x = t, \therefore dx = -dt$

When x = a,t = b

When x = b,t = a

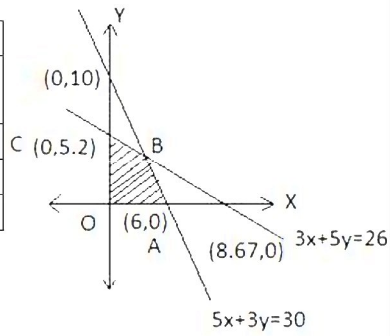
$\therefore$ R.H.S $= \int_b^a f(t)(-dt) = \int_a^b f(t)dt$ ...using $\int_a^b f(x)dx = -\int_b^a f(x)dx$

R.H.S $= \int_a^b f(x)dx$ … … using $\int_a^b f(x)dx = \int_a^b f(t)dt$

Hence proved.

Q.33

Inequalities	Equalities	Points to be plotted
$3x + 5y \le 26$	$3x + 5y = 26$	$\left(\frac{26}{3}, 0\right), \left(0, \frac{26}{5}\right)$
$5x + 3y \le 30$	$5x + 3y = 30$	(6,0)(0,10)
$x \ge 0$	$x = 0$	
$y \ge 0$	$y = 0$	



Shaded portion is the solution set.

Points	Z=7x+11y
O(0,0)	0
A(6,0)	42
B(4.5,2.5)	59
C(0,5.2)	57.2

$\therefore$ z is maximum at **B(4.5,2.5)** and the maximum value is **59**.

Q.34 In $\Delta ABC, If\ m\angle C = 90°$

$\therefore A + B = \frac{\pi}{2}, B = \frac{\pi}{2} - A, \therefore \frac{B}{2} = \frac{\pi}{4} - \frac{A}{2}$

$\therefore tan\frac{B}{2} = tan\left(\frac{\pi}{4} - \frac{A}{2}\right) = \frac{1 - tan\frac{A}{2}}{1 + tan\frac{A}{2}}$ as required.